

The electroweak chiral parameters for a heavy Higgs in the Standard Model*

Maria J. Herrero[†] and Ester Ruiz Morales[‡]

Departamento de Física Teórica
Universidad Autónoma de Madrid
Cantoblanco, 28049 – Madrid, Spain

Abstract

We study the electroweak interactions within the standard electroweak theory in the case where the Higgs particle is heavy, namely, $M_H \leq 1 \text{ TeV}$. By integrating out the Higgs boson to one loop we find the complete effective lagrangian, called electroweak chiral lagrangian, that is $SU(2)_L \times U(1)_Y$ invariant and contains the whole set of operators up to dimension four. The values of the chiral parameters representing the non-decoupling effects of the heavy Higgs are presented. Some examples have been chosen to show the applicability of this effective lagrangian approach to compute phenomenologically relevant quantities that either are being measured at present or will be measured in future experiments.

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[†]e-mail:herrero@vm1.sdi.uam.es

[‡]e-mail: meruiz@vm1.sdi.uam.es

1 Introduction

The electroweak chiral Lagrangian[1, 2] (EChL) provides the most general parametrization for the spontaneous breaking of the $SU(2)_L \times U(1)_Y$ symmetry if the would-be Goldstone bosons (GB) are the only modes of the symmetry breaking sector to be considered at low energies. The GB fields are parametrized non-linearly such that the electroweak chiral Lagrangian built up with these modes and the gauge fields is manifestly $SU(2)_L \times U(1)_Y$ invariant. The price to be paid in this Higgs-less parametrization is that the resulting low energy theory is non-renormalizable, and a tower of effective operators of increasing dimension has to be added at each loop order to render the theory finite.

From the point of view of effective field theories, the electroweak chiral Lagrangian can be regarded as the low energy limit of an underlying fundamental theory, where some heavy fields have been integrated out inducing additional higher dimension operators. These effective operators can, in principle, be determined if the underlying fundamental interactions are known. In a perturbative approach, it is done by explicit calculation of the relevant loop diagrams and by matching the predictions of the full underlying theory (in which heavy particles are present) and those of the low energy effective theory (with only light degrees of freedom) at some reference scale [3]. The matching procedure fixes the value of the effective parameters in terms of the parameters of the full theory, ensuring the equality of the two theories at low energy. This combined picture of integrating out the heavy fields and matching the predictions of the two theories has been recently applied to some particular situations in electroweak interactions [4, 5, 6].

We have derived[5, 7] the EChL that parametrizes electroweak interactions when the underlying theory is the standard model with a heavy Higgs. We deduce the values of the EChL parameters by integrating out the Higgs field to one loop and by matching the standard model predictions in the large M_H limit with the predictions from the chiral Lagrangian to one loop order. By large M_H limit we mean the situation where the mass of the Higgs is much larger than the available external momenta ($p^2 \ll M_H^2$) and other particle masses ($m^2 \ll M_H^2$, $m = M_W, M_Z$), but not so high that perturbation theory is unreliable ($M_H^2 \lesssim 1\text{TeV}^2$).

In section 2 we briefly present the EChL and discuss the renormalization of the effective theory. In section 3, we describe the matching of the standard model with a heavy Higgs and the EChL, and give the corresponding values of the chiral parameters. Section 4 is devoted to discuss some examples of how the EChL formalism can be applied

to calculate electroweak radiative corrections to different phenomenologically relevant quantities.

2 The electroweak chiral Lagrangian

The electroweak chiral Lagrangian based on the symmetry breaking pattern $SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$ is written in terms of the gauge and GB fields [1, 2]

$$\mathcal{L}_{\text{EChL}} = \mathcal{L}_{\text{NL}} + \sum_{i=0}^{13} \mathcal{L}_i, \quad (1)$$

where

$$\mathcal{L}_{\text{NL}} = \frac{v^2}{4} \text{Tr} [D_\mu U^\dagger D^\mu U] + \frac{1}{2} \text{Tr} [\hat{W}_{\mu\nu} \hat{W}^{\mu\nu} + \hat{B}_{\mu\nu} \hat{B}^{\mu\nu}] + \mathcal{L}_{\text{R}_\xi} + \mathcal{L}_{\text{FP}}^{\text{NL}} \quad (2)$$

is the Lagrangian of the gauged non-linear sigma model and the set of $SU(2)_L \times U(1)_Y$ and CP-invariant¹ operators up to dimension four² can be parametrized as

$$\begin{aligned} \mathcal{L}_0 &= a_0 \frac{v^2}{4} [\text{Tr} (TV_\mu)]^2 & \mathcal{L}_7 &= a_7 \text{Tr} (V_\mu V^\mu) [\text{Tr} (TV^\nu)]^2 \\ \mathcal{L}_1 &= a_1 \frac{ig'g'}{2} B_{\mu\nu} \text{Tr} (T\hat{W}^{\mu\nu}) & \mathcal{L}_8 &= a_8 \frac{g^2}{4} [\text{Tr} (T\hat{W}_{\mu\nu})]^2 \\ \mathcal{L}_2 &= a_2 \frac{ig'}{2} B_{\mu\nu} \text{Tr} (T[V^\mu, V^\nu]) & \mathcal{L}_9 &= a_9 \frac{g}{2} \text{Tr} (T\hat{W}_{\mu\nu}) \text{Tr} (T[V^\mu, V^\nu]) \\ \mathcal{L}_3 &= a_3 g \text{Tr} (\hat{W}_{\mu\nu} [V^\mu, V^\nu]) & \mathcal{L}_{10} &= a_{10} [\text{Tr} (TV_\mu) \text{Tr} (TV_\nu)]^2 \\ \mathcal{L}_4 &= a_4 [\text{Tr} (V_\mu V_\nu)]^2 & \mathcal{L}_{11} &= a_{11} \text{Tr} ((D_\mu V^\mu)^2) \\ \mathcal{L}_5 &= a_5 [\text{Tr} (V_\mu V^\mu)]^2 & \mathcal{L}_{12} &= a_{12} \text{Tr} (TD_\mu D_\nu V^\nu) \text{Tr} (TV^\mu) \\ \mathcal{L}_6 &= a_6 \text{Tr} (V_\mu V_\nu) \text{Tr} (TV^\mu) \text{Tr} (TV^\nu) & \mathcal{L}_{13} &= a_{13} \frac{1}{2} [\text{Tr} (TD_\mu V_\nu)]^2 \end{aligned} \quad (3)$$

$$T \equiv U\tau^3 U^\dagger, \quad V_\mu \equiv (D_\mu U)U^\dagger.$$

where $U = \exp \left(i \frac{\vec{\tau} \cdot \vec{\pi}}{v} \right)$ is the GB unitary matrix and $\hat{W}_{\mu\nu}, \hat{B}_{\mu\nu}$ are the field strengths for the $SU(2)_L$ and $U(1)_Y$ gauge fields $\hat{W}_\mu = -i\vec{W}_\mu \cdot \vec{\tau}/2, \hat{B}_\mu = -iB_\mu \tau^3/2$.

¹ There is another CP conserving but C and P violating operator \mathcal{L}_{14} [8]. It is zero in absence of fermionic loop contributions and will not be considered here.

² In a generic R_ξ - gauge, the BRS invariance of the lagrangian implies the existence of operators involving also the ghost fields. These operators, however, are not needed in any calculation of physical quantities at one loop, and therefore we will not include them.

Although this parametrization of electroweak interactions is non-renormalizable in the usual sense, this by no means imply that the predictive power is lost. The electroweak chiral Lagrangian can be properly renormalized at one loop, and all the infinities can be absorbed into redefinitions of the parameters of the lagrangian

$$\begin{aligned}
B_\mu^b &= \widehat{Z}_B^{1/2} B_\mu, & g'^b &= \widehat{Z}_B^{-1/2} (g' - \widehat{\delta g'}), \\
\vec{W}_\mu^b &= \widehat{Z}_W^{1/2} \vec{W}_\mu, & g^b &= \widehat{Z}_W^{-1/2} (g - \widehat{\delta g}), \\
\vec{\pi}^b &= \widehat{Z}_\pi^{1/2} \vec{\pi}, & v^b &= \widehat{Z}_\pi^{1/2} (v - \widehat{\delta v}), \\
\xi_B^b &= \xi_B (1 + \widehat{\delta \xi}_B), & \xi_W^b &= \xi_W (1 + \widehat{\delta \xi}_W), \\
a_i^b &= a_i(\mu) + \delta a_i, & &
\end{aligned} \tag{4}$$

where $\widehat{Z}_i \equiv 1 + \widehat{\delta Z}_i$ and the superscript b denotes bare quantities.

Once a particular renormalization prescription is chosen to fix the counterterms in the effective theory, the renormalized parameters in the right hand side of eq.(4) -that are in general μ -scale and renormalization prescription dependent- remain as free parameters that can not be determined within the framework of the low energy theory. The values of the renormalized chiral parameters $a_i(\mu)$ can be constrained from experiment as they are directly related to different observables in scattering processes[9] and in precision electroweak measurements[10]; but to have any theoretical insight on their values, one has to choose a particular model for the dynamics of the symmetry breaking sector.

3 Chiral parameters for a heavy Higgs

If the underlying fundamental theory is the standard model(SM) with a heavy Higgs, the values of the chiral parameters can be determined by matching the predictions of the standard model in the large Higgs mass limit and those of the EChL at one loop level[5, 7].

We have imposed the strongest form of matching by requiring that all renormalized one-light-particle irreducible (1LPI) Green's functions are the same in both theories at scales $\mu \leq M_H$. This matching condition is equivalent to the equality of the light particle effective action in the two descriptions. In practice, it is enough to analyse the

two, three, and four-point Green's functions with external gauge fields to fix completely the chiral parameters in terms of the parameters of the SM. We have worked in a general R_ξ -gauge to show that the chiral parameters a_i are ξ -independent, and used dimensional regularization to regulate the divergent integrals.

The SM Green's functions are non-local as they depend on p/M_H through the virtual Higgs propagators. When doing the low energy expansion, care must be taken since clearly the operations of making loop integrals and taking the large M_H limit do not commute. Thus, one must first regulate the loop integrals by dimensional regularization, then perform the renormalization with some fixed prescription and at the end take the large M_H limit, with M_H being the renormalized Higgs mass. We have renormalized in the on-shell scheme. From the computational point of view, in the large M_H limit we have neglected contributions that depend on $(p/M_H)^2$ and/or $(m/M_H)^2$ and vanish when the formal $M_H \rightarrow \infty$ limit is taken.

The matching procedure can be summarized by the following relation among renormalized Green's functions

$$\Gamma_R^{\text{SM}}(\mu) = \Gamma_R^{\text{EChL}}(\mu), \quad \mu \leq M_H, \quad (5)$$

where the large Higgs mass limit in the left-hand side must be understood throughout. This equation represents symbolically a system of tensorial coupled equations (as many as 1LPI functions for external gauge fields) with several unknowns, namely the complete set of parameters $a_i(\mu)$ and counterterms that we are interested in determining. There is just one compatible solution given by the following values of the bare electroweak chiral parameters

$$\begin{aligned} a_0^b &= g'^2 \frac{1}{16\pi^2} \frac{3}{8} \left(\Delta_\epsilon - \log \frac{M_H^2}{\mu^2} + \frac{5}{6} \right), \\ a_1^b &= \frac{1}{16\pi^2} \frac{1}{12} \left(\Delta_\epsilon - \log \frac{M_H^2}{\mu^2} + \frac{5}{6} \right), \\ a_2^b &= \frac{1}{16\pi^2} \frac{1}{24} \left(\Delta_\epsilon - \log \frac{M_H^2}{\mu^2} + \frac{17}{6} \right), \\ a_3^b &= \frac{-1}{16\pi^2} \frac{1}{24} \left(\Delta_\epsilon - \log \frac{M_H^2}{\mu^2} + \frac{17}{6} \right), \\ a_4^b &= \frac{-1}{16\pi^2} \frac{1}{12} \left(\Delta_\epsilon - \log \frac{M_H^2}{\mu^2} + \frac{17}{6} \right), \\ a_5^b &= \frac{v^2}{8M_H^2} - \frac{1}{16\pi^2} \frac{1}{24} \left(\Delta_\epsilon - \log \frac{M_H^2}{\mu^2} + \frac{79}{3} - \frac{27\pi}{2\sqrt{3}} \right), \end{aligned}$$

$$\begin{aligned}
a_{11}^b &= \frac{-1}{16\pi^2} \frac{1}{24}, \\
a_6^b &= a_7^b = a_8^b = a_9^b = a_{10}^b = a_{12}^b = a_{13}^b = 0,
\end{aligned} \tag{6}$$

where

$$\Delta_\epsilon \equiv \frac{2}{\epsilon} - \gamma_E + \log 4\pi. \tag{7}$$

We would like to make some remarks on the previous result:

1. First of all, we agree with the $1/\epsilon$ dependence of the a_i^b parameters that was first calculated by Longhitano [2] looking at the divergences of the non-linear sigma model. We see therefore that the divergences generated with the \mathcal{L}_{NL} to one loop are exactly canceled by the $1/\epsilon$ terms in the a_i^b 's.

What is important to realize is that the matching procedure fixes completely the values of the bare parameters a_i^b in terms of the renormalized parameters of the SM. Once we have chosen a particular subtraction scheme to fix the counterterms of the EChL, in our case

$$\begin{aligned}
\delta a_0 &= g'^2 \frac{1}{16\pi^2} \frac{3}{8} \Delta_\epsilon, & \delta a_3 &= \frac{-1}{16\pi^2} \frac{1}{24} \Delta_\epsilon, \\
\delta a_1 &= \frac{1}{16\pi^2} \frac{1}{12} \Delta_\epsilon, & \delta a_4 &= \frac{-1}{16\pi^2} \frac{1}{12} \Delta_\epsilon, \\
\delta a_2 &= \frac{1}{16\pi^2} \frac{1}{24} \Delta_\epsilon, & \delta a_5 &= \frac{-1}{16\pi^2} \frac{1}{24} \Delta_\epsilon,
\end{aligned} \tag{8}$$

then the corresponding renormalized values of the chiral parameters are given in terms of the SM parameters as follows:

$$\begin{aligned}
a_0(\mu) &= g'^2 \frac{1}{16\pi^2} \frac{3}{8} \left(\frac{5}{6} - \log \frac{M_H^2}{\mu^2} \right), & a_3(\mu) &= \frac{-1}{16\pi^2} \frac{1}{24} \left(\frac{17}{6} - \log \frac{M_H^2}{\mu^2} \right), \\
a_1(\mu) &= \frac{1}{16\pi^2} \frac{1}{12} \left(\frac{5}{6} - \log \frac{M_H^2}{\mu^2} \right), & a_4(\mu) &= \frac{-1}{16\pi^2} \frac{1}{12} \left(\frac{17}{6} - \log \frac{M_H^2}{\mu^2} \right), \\
a_2(\mu) &= \frac{1}{16\pi^2} \frac{1}{24} \left(\frac{17}{6} - \log \frac{M_H^2}{\mu^2} \right), & a_{11}(\mu) &= \frac{-1}{16\pi^2} \frac{1}{24}, \\
a_5(\mu) &= \frac{v^2}{8M_H^2} - \frac{1}{16\pi^2} \frac{1}{24} \left(\frac{79}{3} - \frac{27\pi}{2\sqrt{3}} - \log \frac{M_H^2}{\mu^2} \right), \\
a_i(\mu) &= 0, \quad i = 6, 7, 8, 9, 10, 12, 13.
\end{aligned} \tag{9}$$

2. The divergences and the logarithmic running with the scale μ depend only on the $SU(2)_L \times U(1)_Y$ symmetry and therefore, they are the same for any underlying

dynamics for the symmetry breaking sector that has the EChL as its low energy effective theory.

3. Eqs.(6) give the complete non-decoupling effects of a heavy Higgs, that is, the leading logarithmic dependence on M_H and the next to leading constant contribution to the electroweak chiral parameters. The a_i 's are accurate up to corrections of the order (p^2/M_H^2) where $p \approx M_Z$ and higher order corrections in the loop expansion.
4. We demonstrate that the a_i 's are gauge independent, as expected.
5. There is only one effective operator a_5 that gets a tree level contribution. It's value depends on the renormalization prescription that one has chosen in the standard model, on-shell in our case.
6. Only one custodial breaking operator a_0 , which has dimension 2, is generated when integrating out the Higgs at one loop. No custodial breaking operator of dimension four are generated.

We believe that this effective field theory calculation provides valuable information since it clarifies the relation between the linear and non-linear approach to electroweak interactions. The chiral parameters of eqs.(6) also serve as reference values to be compared with the corresponding predictions from other possible alternatives for the symmetry breaking.

4 Calculating observables with the EChL

In this section we will show, as an example, the explicit calculation of the radiative corrections to Δr within the electroweak chiral Lagrangian approach. In the on-shell scheme, Δr is defined as[11]

$$\begin{aligned} \Delta r = & \Sigma'_{\gamma\gamma}(0) + \frac{c_w^2}{s_w^2} \left(\frac{\Sigma_{WW}(M_W^2)}{M_W^2} - \frac{\Sigma_{ZZ}(M_Z^2)}{M_Z^2} + \frac{2s_w}{c_w} \frac{\Sigma_{\gamma Z}(0)}{M_Z^2} \right) \\ & + \frac{\Sigma_{WW}(0) - \Sigma_{WW}(M_W^2)}{M_W^2} + \frac{\alpha}{4\pi s_W^2} \left(6 + \frac{7 - 4s_W^2}{2s_W^2} \log c_W^2 \right) \end{aligned} \quad (10)$$

where Σ denote unrenormalized self-energies, and therefore will have the contribution from the loop diagrams generated with \mathcal{L}_{NL} and contribution from the bare a_i^b effective

parameters. Considering only bosonic contributions, the loop diagrams give³

$$\Delta r|_{\text{loops}}^{\text{bosonic}} = \frac{g^2}{16\pi^2} \left[\frac{11}{12} \Delta_\epsilon - \frac{11}{12} \log \frac{M_W^2}{\mu^2} + a(M_W^2, M_Z^2) \right] \quad (11)$$

where $a(M_W^2, M_Z^2)$ include the finite and scale-independent contribution from the EChL loops to Δr

$$\begin{aligned} a(M_W^2, M_Z^2) = & \log c_W^2 \left(\frac{5}{c_W^2} - 1 + \frac{3c_W^2}{s_W^2} - \frac{17}{4s_W^2 c_W^2} \right) - s_W^2 (3 + 4c_W^2) F(M_Z^2, M_W, M_W) \\ & + I_2(c_W^2) \left(1 - \frac{c_W^2}{s_W^2} \right) + \frac{c_W^2}{s_W^2} I_1(c_W^2) + \frac{1}{8c_W^2} (43s_W^2 - 38) \\ & + \frac{1}{18} (154s_W^2 - 166c_W^2) + \frac{1}{4c_W^2} + \frac{1}{6} + \left(6 + \frac{7 - 4s_W^2}{2s_W^2} \log c_W^2 \right). \end{aligned} \quad (12)$$

The contribution to Δr coming from the effective operators can be easily obtained from their corresponding contribution to the self energies given in [5]

$$\Delta r|_{a_i} = -2g^2 a_1^b - 2 \frac{c_W^2}{s_W^2} a_0^b = -2g^2 a_1(\mu) - 2 \frac{c_W^2}{s_W^2} a_0(\mu) - \frac{g^2}{16\pi^2} \frac{11}{12} \Delta_\epsilon, \quad (13)$$

where we have taken the subtraction scheme defined in eqs.(8) for the effective chiral parameters.

Finally, the expression for Δr in the EChL formalism is

$$\Delta r^{\text{EChL}} = -2g^2 a_1(\mu) - 2 \frac{c_W^2}{s_W^2} a_0(\mu) - \frac{g^2}{16\pi^2} \frac{11}{12} \log \frac{M_W^2}{\mu^2} + \frac{g^2}{16\pi^2} a(M_W^2, M_Z^2). \quad (14)$$

The divergences of the bosonic loop contributions have been cancelled by the δa_0 and δa_1 counterterms. The μ -scale dependence of the loop contributions is also properly cancelled by the scale dependence of the renormalized $a_i(\mu)$'s, so that the observable is μ -scale and renormalization prescription independent.

In the particular case of the SM with a heavy Higgs, one has just to substitute the values of $a_i(\mu)$ from eqs.(9) to obtain

$$\Delta r_{\text{Higgs}}^{\text{heavy}} = \frac{g^2}{16\pi^2} \left(\frac{11}{12} \log \frac{M_H^2}{M_W^2} - \frac{5}{6} \right) + \frac{g^2}{16\pi^2} a(M_W^2, M_Z^2). \quad (15)$$

which agrees with the result given in [11].

One can similarly obtain the heavy Higgs contributions to the rest of electroweak parameters[5, 7].

³See the second paper in ref.[10]

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